

Validation of a simple relationship between the fracture pattern and the fracture stress of glass

Marco ZACCARIA*, Mauro OVEREND^a,

* Glass & Façade Technology Research Group
University of Cambridge
Trumpington Street, CB2 1PZ, Cambridge (UK)
mz287@cam.ac.uk

^a Glass & Façade Technology Research Group
University of Cambridge

Abstract

Several studies show that a relationship between the first branch length of a glass component and the stress at failure is: $\sigma_f = \alpha r^{-1/2}$. This can be used to determine the fracture stress of a glass component knowing the length of the first branch by means of a branching constant. However the fracture pattern is not always clear, making the branch length difficult to measure. Furthermore it is unclear whether macroscopic flaws have any effect on the branching relationship. This paper makes a contribution to both these issues by explaining the procedure adopted to read the fracture features and by investigating both weathered and as-received glass. This study endeavours to provide useful techniques for the rapid diagnostic interpretation of glass failure.

Keywords: crack branching, fractography, surface flaws, fragmentation.

1 Introduction

Designing with glass is as complex as fascinating is the outcome. Despite the considerable amount of research that has been carried out to improve its mechanical properties glass remains a brittle material and its tensile strength cannot be considered a material constant. The flaws due to manufacturing, handling, transport, weathering and its general use cause stress concentrations that may lead to under or over estimation of its design strength. Therefore an accurate approach requires knowledge of fracture mechanics properties such as fracture toughness and slow crack growth which are able to account for flaws and stress concentrations. Recent studies show how the tensile strength of glass can be determined explicitly or stochastically [1]. However a simple relationship between the stress at failure and the fracture pattern could be a useful tool to obtain quantitative information after failure. Although such a relationship, called the crack branching equation, is generally

accepted [2, 3], its application is not always straightforward because reading the fracture pattern becomes quite complex when high stresses are relieved at failure. Furthermore a standard procedure for a fractographic inspection has yet to be defined. This shortcoming extends to specialized standards such as ATMS C 1322 [4], which fails to provide any guidelines on this issue.

This paper presents a series of 4-point bending (4PB) and coaxial double ring (CDR) tests of glass beams and panes to evaluate the crack branching equation and to establish a value for the branching constant. Both fused silica glass and soda-lime glass were used. The latter was both new (as-received) and weathered (i.e. 20 years old window). In this way it was possible to compare the results and evaluate any effects from the weathering.

2 Crack branching theory

Due to its flaws glass could be modelled as a cracked body. The study of a cracked body was first introduced by Griffith [5]. A formulation to model the behaviour in such a case was derived by Irwin [6], who introduced the concept of the stress intensity factor. The latter is a measure of the stress concentration near the crack tip and for mode I (pure tensile stress) is defined as:

$$K_{IC} = Y \sigma_n \sqrt{\pi a} \quad (1)$$

Where Y is a geometry factor, which depends on the geometry of the crack (see [4] for values), σ_n is the nominal tensile stress normal to the crack's plane and a represents the depth of the crack. In terms of stress intensity factor, failure occurs when:

$$K_I > K_{IC} \quad (2)$$

Where K_{IC} is generally accepted to be a material constant equal to $0.75 \text{ MPa m}^{1/2}$ for soda-lime glass. [7].

However, another phenomena known as slow crack growth could lead to fracture even if $K_I < K_{IC}$. Definitions and detail of slow crack growth are beyond the scope of this paper. Readers should refer to Munz and Fett [8], Fuller and Wiederhorn et al. [9], and Overend and Zammit [1] for a comprehensive explanation. When $K_I \geq K_{IC}$ the system loses equilibrium catastrophically: crack propagation occurs instantaneously. A first explanation of crack branching was indeed attributed to a crack propagation process. In other words, the hypothesis put forward by Shetty et al. [10] was that the crack accelerates until a maximum speed is reached, at which the crack bifurcates to dissipate energy [10]. This theory appears not to have been proven experimentally.

However, from an energy point of view, it is obvious that the energy available in the system must be dissipated through bulk material deformation and material separation. Indeed, considering the Mott energy balance [11], as an extension of Griffith's:

$$U = U_M + U_S + U_K \quad (3)$$

where, U is the total energy in the system, U_M the mechanical energy, U_S the force energy expended in creating new crack surfaces, U_K the kinetic energy. The higher the strain energy in the material, the more crack surfaces will be created. Velocity comes into effect as the instantaneous acceleration of the crack could be the cause of the formation of a featured area around the crack origin. Observing the crack origin with a microscope it is possible to recognize three different areas: a smooth surface, called mirror, a slightly roughened area, called mist and a severely roughened area, called hackle (Fig. 1). After the hackle is created crack bifurcates generating the branching. An explanation of the branching is still elusive, however this empirical relationship, called the crack branching equation is generally accepted [2], [3]:

$$\sigma_f = \frac{\alpha}{\sqrt{r}} \quad (4)$$

where, σ_f is the stress at failure, r is the mirror/mist/hackle radius or branch length and α is mirror/mist/hackle/branch constant.

The ensuing parts of this paper explain the procedure followed to validate the equation, as well as the shortcomings of its use.

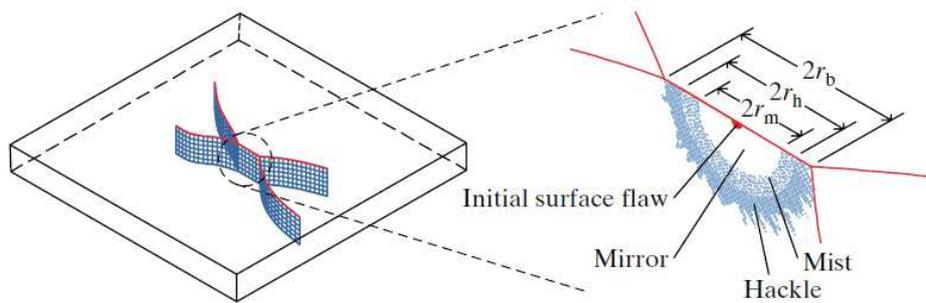


Figure 1: Schematic representation of mirror, mist, hackle and branching. Courtesy of [7].

3 Test procedure

CDR and 4PB tests were performed on glass specimens, from which the fracture mirror and crack branching were located and measured. These were in turn used to calculate the mirror and branching constant respectively. Soda-lime glass either new (i.e. as-received) or weathered in 300x300x3 mm panes were tested on the CDR jig, whereas fused silica glass specimens were tested either on the CDR or 4PB jigs (66x66x3 mm panes for the CDR and 45x4x3 mm for the 4PB). The different load rates and size of the specimens used, do not affect the crack-branching results.

LUSAS FEM software was used to determine the stress state of the CDR specimens, whereas for the 4PB tests, simple beam theory was applied. In order to keep the pieces together an adhesive film was attached to the compressive side. This facilitated the reading of fracture features after failure.

For the CDR specimens the crack branch length was determined with the naked eye and digital photographs. On the other hand, given the small size of the 4PB specimens, the fracture mirror was the preferred feature for the 4PB specimens and was measured by means of optical microscopy.

3.1 4-Points bending tests

Fused silica specimens were polished with an 80-50 dig designation [12], whereas the edges were chamfered but not polished. Therefore, most of the failures occurred from the edges rather than from the surface.

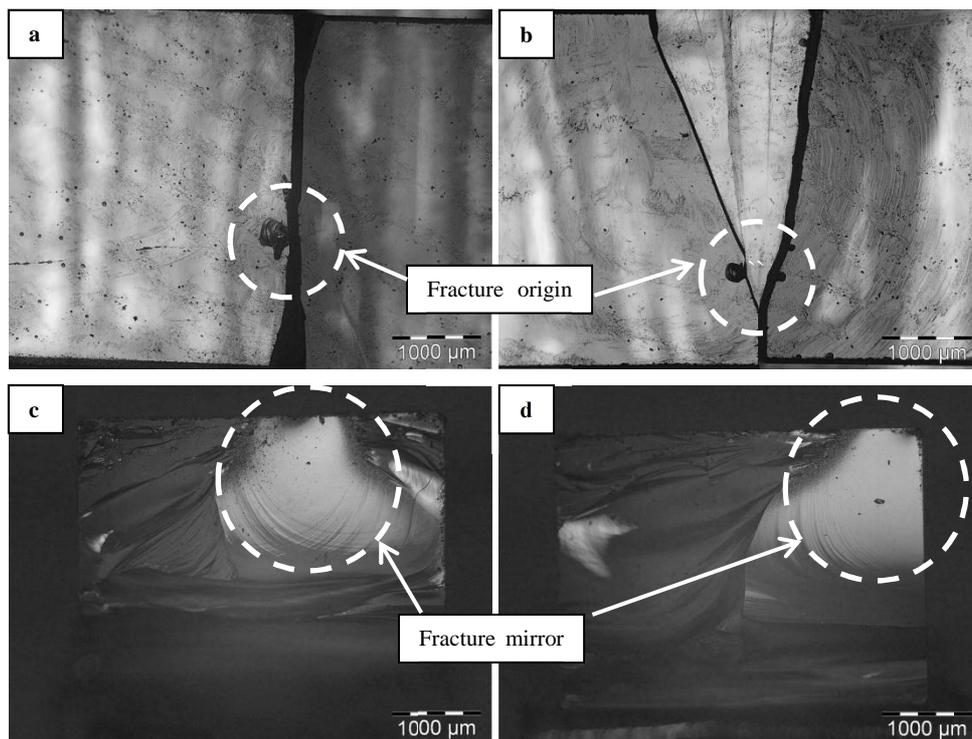


Figure 2: 4PB fused silica specimens a) top view of a surface fracture origin; b), top view of an edge fracture origin; c) fracture mirror of the specimen in 2a); d) fracture mirror of the specimen in 2b).

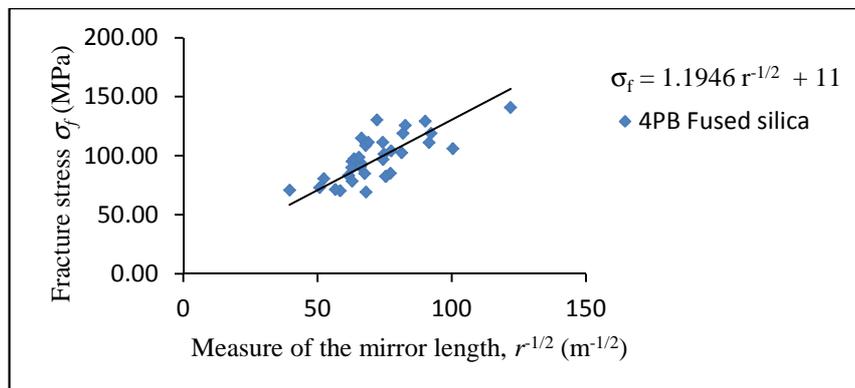
Figure 2 shows the fracture origin on the surface and the relevant cross section for both edge and surface failure.

From the 40 fused silica specimens tested it was possible to read the mirror on 33 of the samples. Whenever possible the readings, were done on both sides of the broken samples and in most cases the measurements of the two mirrors matched very well.

Plotting the data (Fig. 3) and using a linear regression it can be seen that there is good agreement ($R^2 = 0.834$) between the slope of the curve and the average value of the mirror constant. However intercept is not zero, but is approximately -11 MPa. This could be explained by the presence of a surface residual stress on the glass. This explanation has already been put forward by Conway and Mecholsky [13] and on-going work by the author involves reading the surface residual stress using photoelasticity in order to validate or correct the equation with the right intercept. Therefore the equation in that case would be:

$$\sigma_f = \frac{\alpha}{\sqrt{r}} + f_i \quad (5)$$

where f_i is the residual surface strength, equal and opposite to the σ_i .



Average Stress σ_f (MPa)	Average half Mirror r (m)	Mirror Constant α (MPa $m^{1/2}$)	Residual stress σ_i (MPa)
98	2.18×10^{-4}	1.37	-11

Figure 3: fitting of the fused silica results with a linear regression and mean values.

3.2 Ring-on-ring results

New and weathered glass (20 years old windows), in 300x300x3 mm panes were tested. Failure occurred mainly within or very close to the loading ring. The crack branching in the weathered glass was generally easier to read, as the stress at failure was lower. Despite a total of 200 specimens (100 weathered, 100 as received) were tested, the crack branching was clear on only 32 (20 weathered, 12 new) of the specimens (fig. 4). The crack branching readings were even more problematic in the fused silica specimens, as in general the stress at failure was higher, therefore resulting in a higher density of fragments.

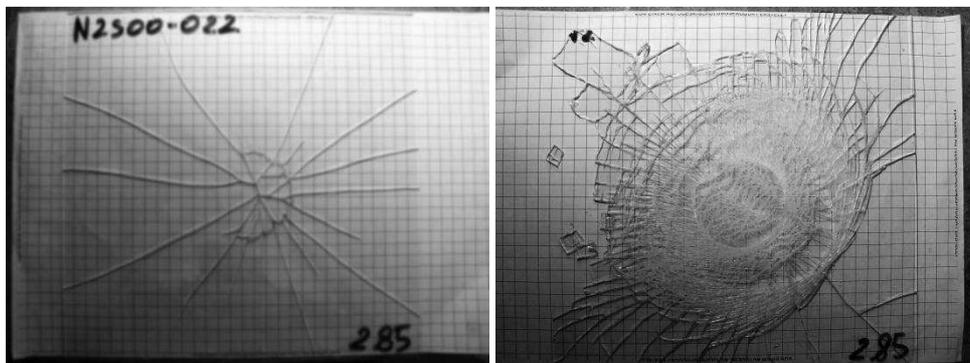
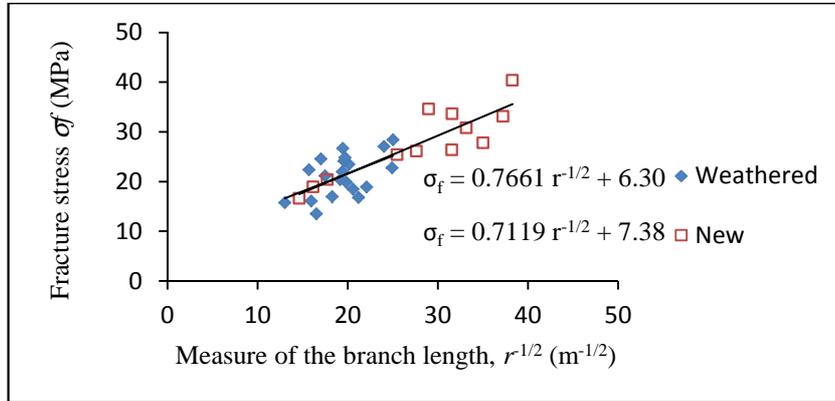


Figure 4: Left: large fragments, clear branching on a CDR test; Right: high density of fragments, unclear branching.

There was a negligible difference in the slope (i.e. the branching constant) between the new and the weathered soda-lime glass (Fig. 5), and the value of the residual stress was also very similar. A larger difference can be observed between the fused silica and the soda-lime, where the former exhibits higher value of the branching constant, and a higher value of the residual stress, although further data is required to validate this.



Glass type	No. of Specimens	Average Stress σ_f (MPa)	Average half Mirror r (m)	Branching Constant α (MPa m ^{1/2})	Residual stress σ_i (MPa)
Weathered	20	21	2.85×10^{-3}	1.10	-7
New	12	28	1.74×10^{-3}	1.01	-6
Fused Silica	7	91	8.1×10^{-4}	2.37	-6

Figure 5: Fitting of the CDR test of new and weathered glass with a linear regression and mean values.

4 Discussions

One of the purposes of this study was to establish a standard procedure for measuring the crack features, as suggested by Quinn [2] and ASTM C 1322 [4]. Indeed the smaller fragments size generated with high stress on the CDR were found to cause several difficulties for the fractographer, principally to determine the branch length when the density of fragmentation is high. It has yet to be established whether all the fragmentation occurs through dynamic crack growth at the instant of first fracture or whether some of the fragmentation occurs at a subsequent stage in response to compatibility of deformations. The use of a high speed camera would be beneficial to establish which of the two hypotheses is correct. In the 4PB tests, measuring the mirror is a straightforward process, as the mirror feature is easily identified. However, when stress at failure is high there is an increased risk that the fragment containing the mirror is propelled from the specimen on fracture and lost.

To sum up, from a fractography point of view it is important to establish the origin of failure in the first instance. Then depending on the type of test and on the fracture pattern it is advisable to either identify the mirror features with an optical microscope or to determine the crack branch length with the naked eye. When the stress at failure is quite high ($\sigma_f > 30$ MPa for soda-lime glass, with a residual stress $\sigma_f \approx 7$ MPa) there is a risk that due to the high density of fragments none of these features will be spotted.

The second purpose of this study was to determine whether surface flaws caused by weathering had any effect on the branching characteristics.. The strength data showed that weathered glass is considerably weaker than new glass, but the crack branching constant remains unchanged even for the severely weathered soda-lime glass investigated. Furthermore, as the fracture stress of weathered glass and the resulting fragmentation density are generally lower, the equation is easier to implement in weathered glass.

5 Conclusions

The crack branching equation seems to be a reliable approach for estimating the fracture stress of a broken glass. Although there is a shortcoming with the reading of such a fracture feature which limits its application to relatively low fracture stresses (≤ 30 MPa). The value of the constant obtained (Fig. 5) differs from the existing literature ($\alpha = 2.18 \text{ MPa m}^{1/2}$ [7]), therefore further tests involving larger populations of specimens are recommended.

The study has also indicated the presence of a residual stress in soda-lime glass, that is often ignored in fracture mechanics. This is the subject of an on-going study by the authors and involves validating the residual stresses obtained from the branching equation with direct photoelastic measurements.

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7 References

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