Paper

Recent developments in design methods for glass structures

M. Overend, BE&A, MSc, PhD, CEng, MIStructE, MICE
Lecturer in Building Engineering Design, University of Cambridge, UK.

Keywords: Glass, Design, Structural members, Loads, Strength, Testing
Received: 05/09: **Modified:** 11/09; **Accepted:** 12/09

Synopsis

The use of glass in buildings has undergone a rapid transformation over the last few decades. Its traditional use as an infill panel is still popular, but an alternative structural use of glass has emerged in which glass elements contribute to the overall load bearing capacity of the structure or sub-structure. The original rules-of-thumb for sizing infill glass panels are largely inadequate for this new generation of glass structures and a range of more rigorous design methods and prototype testing techniques are becoming available. This paper presents an overview of the design process for structural glass elements with an emphasis on analytical techniques and prototype testing methods for determining the tensile strength of glass elements. The paper also describes a novel stress-history interaction equation that is useful for determining the tensile strength of glass with accuracy and ease.

Introduction

Glass is a ubiquitous material and its unique combination of transparency, durability, low cost and high quality finish, made possible by the invention of the float process in the 1950s, has fuelled its popularity over the last century. The subsequent improvements to the tensile strength of glass by heat treatment and the development of discrete connections such as clamped fixings and bolted connections helped to fulfil the architectural vision of the transparent building envelope. There are several recent examples of this 'engineered transparency' in building envelopes such as the Reinbach Pavilion (Fig 1) and the post-tensioned cable façade at the Kempinski Hotel in Munich (Fig 2).

The use of glass is however not confined to the building envelope and there are several outstanding examples of how structural glass engineering has been adopted in and around buildings, such as in floors, canopies (Fig 3) and staircases (Fig 4).

The removal of the opaque structural elements that traditionally supported the glass panels has given way to 'structural glass' where the glass contributes to the load bearing capacity of the larger assembly or structure. This load bearing role may in turn be subdivided further into primary and secondary load bearing glass elements.

Unlike other established load bearing materials, the structural design methods for glass are still in their infancy, but on-going research in this area is rapidly redressing this. There are a few general sources of information such as CPD courses, the Institution of Structural Engineers guidebook[1] and the recent structural engineering document published by IABSE[2], however these need to be updated regularly due to the rapid progress in glass engineering. One of the largest gaps in information is in the selection of an appropriate design method, whereby questions such as: 'Is an empirical calculation sufficient or should I undertake more detailed calculations and / or prototype testing?' remains largely unanswered.

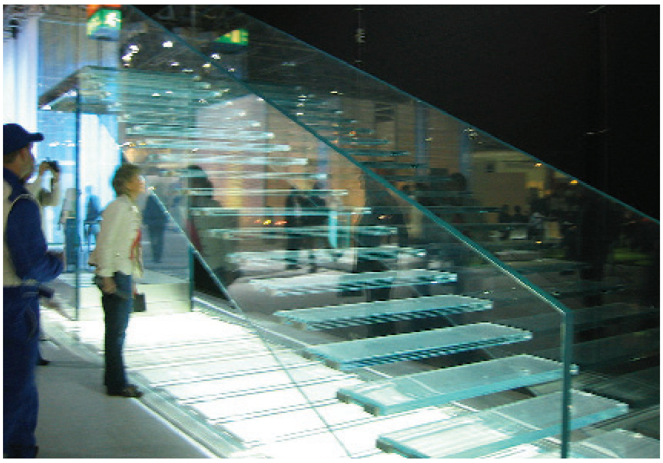
This paper aims to redress this issue by reviewing and extending the principal methods for determining the strength of glass. The paper starts by mapping out the design process and structural performance requirements for glass elements. This is followed by a review of the mechanical properties that underpin structural behaviour, and how these fundamental material properties affect the tensile strength and post-fracture performance. The paper describes how this knowledge may be deployed in design methods, in particular, how multiple load combinations may be taken into account through a simple stress history interaction equation. The paper concludes with an overview of the use of prototype testing in structural glass design. Eurocode terminology and notation is used wherever possible.

1

2

1 Load bearing glass walls at the Reinbach pavilion, Germany. (Sedlacek & Partner)
2 Post-tensioned cable net façade with glass panels at the Kempinski Hotel, Munich, Germany (Schlaich Bergermann)

3



4

3 **Yurakucho glass canopy, Tokyo.** *(Dewhurst Macfarlane)*

4 **Glass staircase in Glasstec Exhibition, Düsseldorf, Germany** *(Seele)*

Function of glass element	Partial actions and combination factors										
	γ_G	γ_Q	ψ_0			ψ_1			ψ_2		
			live†	wind	snow*	live†	wind	snow*	live†	wind	snow*
Primary structure	1.35 (1.0)	1.5 (0)	0.7	0.6	0.5	0.7	0.2	0.2	0.6	0	0
Secondary structure	1.2 (1.0)	1.3 (0)	0.7	0.6	0.6	0.7	0.9	1.0	0.6	0.2	0.2
Infill panel	1.0 (1.0)	1.1 (0)	0.7	0.6	0.9	0.7	0.8	1.0	0.6	0.2	0.2

†Shopping and congested areas. For other building categories refer to EN 1990
() Partial factor for favourable action
*CEN member states except Finland, Iceland, Norway, Sweden and H ≤ 1000m a.s.l.

Table 1 Partial factors for actions and combinations

Design combination	Load duration	Load duration factor. k_{mod}
Long term combination, F_{dL} e.g. self weight	$t_f > 6$ weeks	0.29
Medium term combination, F_{dM} e.g. sustained imposed loads, seasonal temperature, snow and self weight	6 weeks $\geq t_f >$ 10 mins	0.43
Short term combination, F_{dS} e.g. wind, access loads, sustained imposed loads, wind, temperature, snow and self weight	$t_f \leq$ 10 mins	0.74

Table 2 Load duration combinations proposed by the draft European standard[4]

The design process and performance requirements for glass elements

The overall design procedure for structural glass elements is similar to that adopted in other structural materials, and consists of an iterative process for selecting an efficient design that satisfies predetermined performance requirements.

The decision on whether empirical calculations are sufficient or whether they should be supported by more detailed calculations and validated by prototype testing depends on the engineer's confidence levels in his/her calculations. For instance a bespoke glass structure with novel connections would generally require a greater extent of calculations and prototype testing than a standard application that has stood the test of time.

The full design process relies on a combination of the following:

- Rules-of-thumb, used at early design stage to test alternative schemes and as quick checks at a later stage.
- Accurate analytical / numerical methods, employed during detailed design stages.
- Prototype testing, used to validate designs prior to construction, particularly where a novel structure or a new analytical method is used.

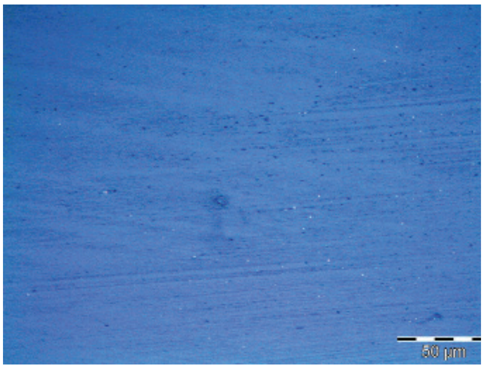
Ultimate Limit State (normal use)

The aim here is to ensure adequate strength and stability for normal use, construction stage actions and routine maintenance. The fundamental combinations of permanent and transient actions may be determined from:

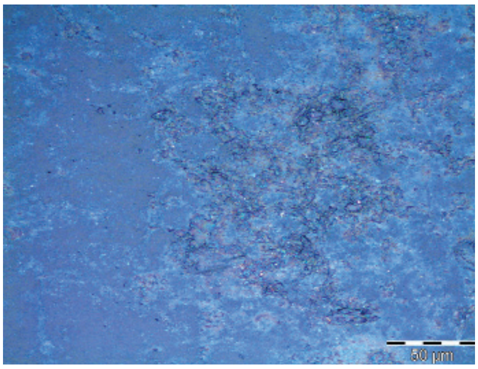
$$F_d = \gamma_G G \text{ " + "} \gamma_Q Q_{k,1} \text{ " + "} \gamma_Q \sum_i \psi_{0,i} Q_{k,i} \qquad \text{...(1)}$$

Table 1 shows a summary of partial load factors obtained by combining the recommendations in EN1990[3] and the draft European standard on glass in building[4]. The combination factors for glass structures are still under discussion. This explains some of the anomalies in this table. For example, the combination factors for primary structural elements appear to be lower than those for secondary structural elements and infill panels. Furthermore the combination factors for frequent value actions, ψ_1, are uncharacteristically larger than the combination factors for non-dominant actions ψ_0. This table, particularly the combination factors ψ, should therefore be used with caution.

Glass is sensitive to stress corrosion. Complete action history models are therefore required in order to design glass elements accurately. A simple way to describe complex real-world stress histories is to use Equation (1) to generate three design combinations for short (F_{dS}); medium (F_{dM}); and long (F_{dL}) term actions respectively as shown in Table 2. It is important to note that the short term combination F_{dS}, also includes long and medium term actions (such as self-weight and imposed loads) which are also present during the 10min time period. Likewise the medium term combination F_{dM}, also includes the long term actions The significance and use of the stress duration factor, k_{mod}, is discussed in subsequent sections of this paper.

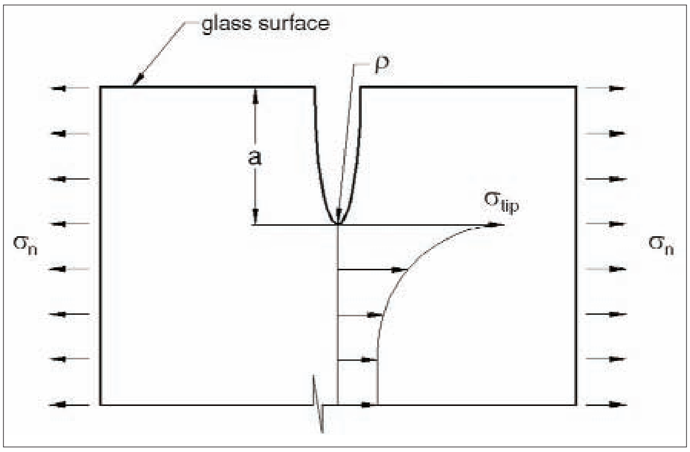


5a

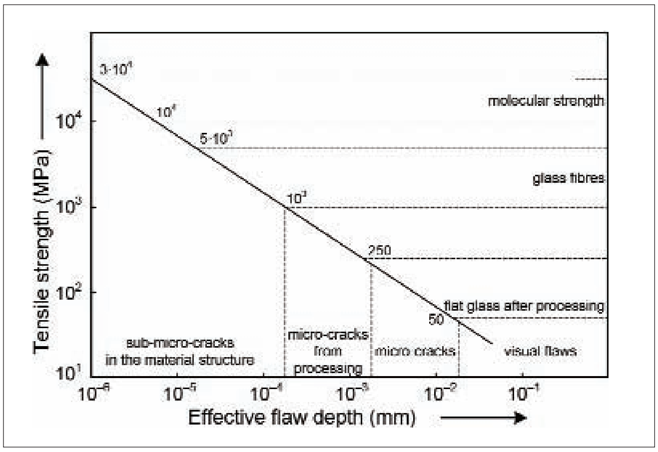


5b

5 Surface of glass viewed through optical microscope showing (a) protected surface (left) and (b) weathered surface (right).
6 Stress intensity at flaw tip
7 Short-term tensile strength as a function of flaw depth[2]



6



7

Ultimate Limit State (exceptional conditions)

It is often impossible or uneconomic to prevent fracture of glass in accidental design situations. Under these conditions, the aim is to ensure safe failure or adequate residual post-fracture capacity of the glass elements. The combinations for accidental design situations may be determined from:

$$F_d = G\text{ " + "}A_d\text{ " + "}\psi_1 Q_{k,1}\text{ " + "}\sum_i \psi_{2,i} Q_{k,i} \qquad ...(2)$$

Exceptional loading conditions may vary considerably from one project to another and it may therefore be necessary to undertake a rigorous risk analysis[5]. The residual post fracture capacity may be determined by assessing the effect of the permanent and transient loads on the fractured glass after the accidental event has passed i.e. the failed structure is subjected to a post-breakage design load of $F_d - A_d$ in equation 2.

Serviceability Limit State

Serviceability limit state requirements include limiting deflections and/or vibrations, to ensure the functioning and the appearance of the structure as well as the comfort of the users.

The design combinations for serviceability may be obtained from:

$$F_d = G\text{ " + "}\psi_1 \cdot Q_{k,1}\text{ " + "}\sum_i \psi_{2,i} Q_{k,i} \qquad ...(3)$$

Basic manufacture and design strength

Molecular structure and surface flaws

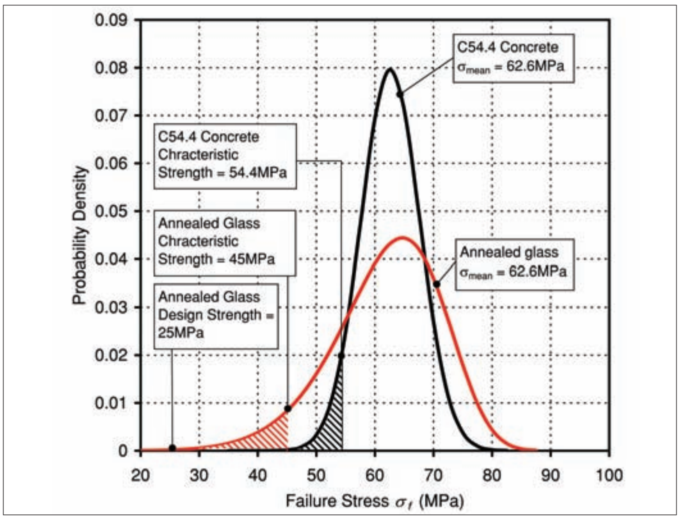
Ninety percent of the glass production world-wide consists of soda-lime-silica glass manufactured by the float process[2, 6, 7]. As the glass cools rapidly from 1100°C to 800°C in the float bath its viscosity increases to approximately 10^{14}Pa s effectively becoming a solid wherein randomly oriented molecules form an amorphous isotropic material with no slip planes or dislocations to allow yield before fracture. Consequently, glass exhibits almost perfectly elastic, isotropic behaviour and brittle fracture. Unlike other construction materials such as steel, glass does not yield plastically and is therefore unable to redistribute stress concentrations by yielding locally. The theoretical tensile strength of annealed glass, based on molecular forces, is exceptionally high and may reach 32GPa. However, the actual tensile strength is several orders of magnitude lower. The reason for this discrepancy is the presence of stress raising flaws on the surface of the glass, first observed by Griffith in 1920[8]. The surface flaws arise from manufacturing, handling, weathering and malicious attack. Fig 5 shows magnified surface images of 20-year old glass. These images reveal the significant damage accumulation on the outer facing (weathered) surface of the glass relative to the inner facing (protected) surface. The stress magnification at the tip of a typical elliptical flaw (Fig 6) may be represented analytically by:

$$\sigma_{tip} = 2\sigma_n\sqrt{a/\rho} \qquad ...(4)$$

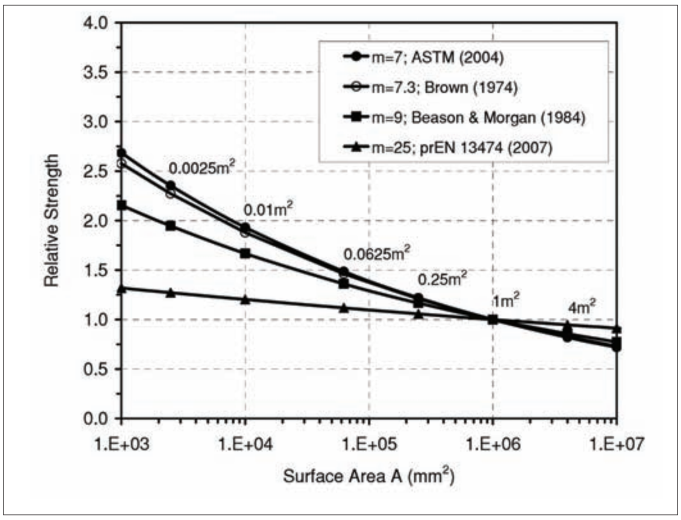
A crack will propagate in the glass when the stress intensity at the tip of one flaw exceeds the molecular strength. Although the flaw depth (or half width), a, at the critical flaw may be less than 1mm, the radius of curvature at the flaw tip, ρ, is less than 1×10^{-9}mm, which explains why the observed tensile strength of glass is several orders of magnitude lower than the theoretical molecular strength. Fig 7 shows the variation of short-term tensile strength with flaw depth.

Cracks are unable to propagate in the presence of compression; as a result the compressive strength of glass is much larger than the tensile strength. The compressive strength is however irrelevant for structural applications as indirect tensile stresses arsing from Poisson's ratio effects or from buckling will dominate the design.

Irwin[9] extended the original Griffith energy-balance concept to characterise a material in terms of its brittleness or fracture toughness. These formulations may be used to determine the fracture strength of glass for known flaw geometries. A detailed review of this approach is available in Haldimann[10] and summarised in Haldimann *et al*[2]. Deterministic fracture-mechanics-



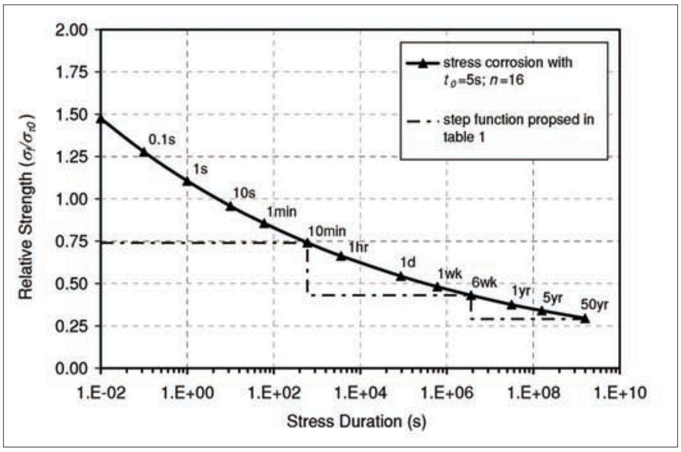
8



9

Source	Surface Strength Parameters		P_f	$f_{gPf,t60}$ (MPa)
Brown (1974); *as-received glass*	m = 7.3	k = 5.1x10^{-57}m^{-2} Pa$^{-7.3}$	1/125 1/1000	15.5 11.9
Beason (1980) *weathered glass*	m = 6	k = 7.19x10^{-45}m^{-2} Pa^{-6}	1/125 1/1000	10.2 7.2
Beason & Morgan (1984) *as-received glass*	m = 9	k = 1.32x10^{-69}m^{-2} Pa^{-9}	1/125 1/1000	26.3 20.9
ASTM (2004); *weathered glass*	m = 7	k = 2.86x10^{-53}m^{-2} Pa^{-7}	1/125 1/1000	16.1 12.0
prEN13474 (2007) *as-received glass*	m = 25	k = 2.35x10^{-188}m^{-2} Pa^{-25}	1/125 1/1000	14.4 13.4

Table 3 Surface strength parameters and corresponding 60s tensile strength



10

based calculations are however of limited use in real-world design applications unless the severity and distribution of the flaws are known. This is rarely the case and flaw characteristics vary considerably from one glass element to another. For most structural design purposes it is therefore more convenient to express the strength of glass statistically in terms of:

- surface condition (i.e. severity and distribution of surface flaws);
- surface area exposed to tensile stress;
- surface stress history (i.e. magnitude and duration);
- environmental conditions (especially humidity).

Surface condition

A large scatter of strength values is always obtained when a batch of nominally identical test pieces of a glass are broken in a carefully controlled way. This dispersion is a result of the variations in surface flaw characteristics and may be represented by a 2-parameter Weibull distribution[11, 12].

$$P_f = 1 - \exp\left(-kA\sigma_f^{\,m}\right) \qquad \text{...(5)}$$

Where P_f is the probability of failure and m and k are two interdependent parameters whose typical values are shown in shown in Table 3.

The surface strength parameters adopted by the most recent version of the draft European Standard[4] are based on the Damage Equivalent Load Resistance (DELR) method[13]. The corresponding probability density function is plotted in Fig 8. For comparison, this is superimposed on a typical probability density function for concrete that has an identical mean compressive strength to the mean tensile strength of glass annealed glass. This is a purely qualitative comparison as the concrete strength is plotted on the positive (tension) scale, but it nevertheless is an effective illustration of the variability of glass and particularly the influence of this variability on the characteristic strength of the material.

8 Probability density functions for annealed glass and concrete
9 Tensile strength as a function of surface area
10 Relative tensile strength ($\sigma_f/\sigma_{t0} = k_{mod}$) vs. stress duration

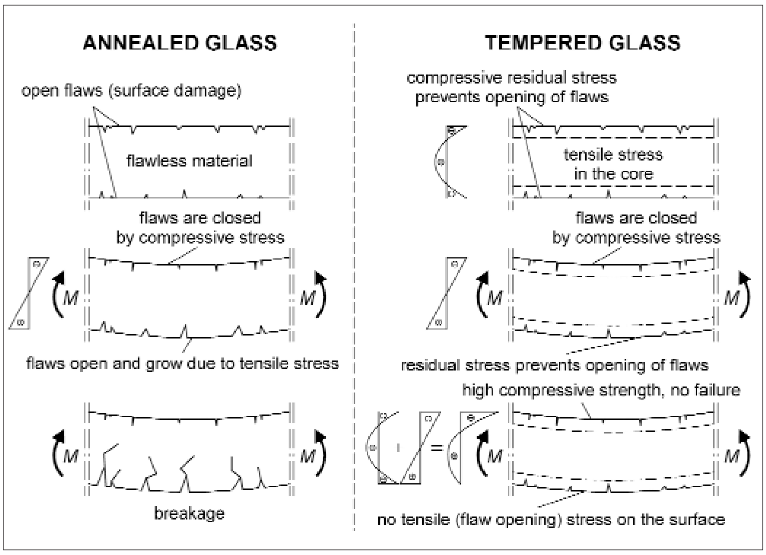
Surface area

The probability of encountering a critical flaw in a glass plate increases with larger surface areas. A large glass plate is therefore statistically weaker than a smaller one. This phenomenon commonly referred to as 'size effect' is expressed in Equation 6 and is plotted in Fig 9.

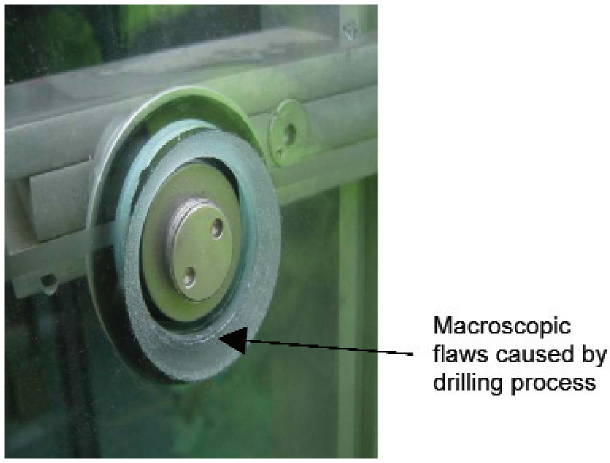
$$\frac{P_{f,A}}{P_{f,A_0}} = \left(\frac{A}{A_0}\right)^{1/m} \qquad \text{...(6)}$$

Stress history and environmental conditions

When loaded in a vacuum, the stress intensity at any flaw tip may either cause fast fracture (where the crack propagates at approximately 1500ms^{-1}) or not grow at all. In such conditions, the strength of glass is independent of time and is governed by the plane strain fracture toughness. In the presence of humidity however, an intermediate phenomenon is triggered wherein the flaws grow sub-critically (1m/s and 0.001m/s) on exposure to a crack opening stress. This is known as stress corrosion (or static fatigue) and is relevant to the structural use of glass as it causes a reduction of the tensile strength of a loaded glass element at atmospheric conditions with time. It was first observed in 1899[13] and can be expressed by Brown's integral[2, 14] that is convenient for converting a time-varying stress $\sigma_f(t)$ applied over a duration t_f to an equivalent uniform stress σ_{t0} applied for a reference period t_0. For constant environmental conditions this involves integrating the tensile stress $\sigma(t)$ (raised to the static fatigue constant n) over the



11



12

11 The benefits of heat treating glass showing (a) Annealed glass and (b) Tempered glass[3]
12 Hole edge damaged by drilling process (protective cap removed for inspection)

stress duration t_f, as shown in equation 7:

$$\sigma_{t0} = \left[\frac{1}{t_0}\int_0^{t_f} \sigma_f{}^n(t)dt\right]^{1/n} \approx \left[\frac{1}{t_0}\sum_{j=1}^{J}(\sigma_f{}^n t_j)\right]^{1/m} \quad ...(7)$$

When a constant stress is applied for a duration t_f equation 7 reduces to:

$$\frac{\sigma_f}{\sigma_{t0}} = \left(\frac{t_0}{t_f}\right)^{1/n} = k_{\text{mod}} \quad ...(8)$$

The σ_f/σ_{t0} ratio is commonly known as the stress corrosion ratio and represented by k_{mod}. It is a useful measure of relative strength for different load durations and is shown in Fig 10. The static fatigue constant, n, is a function of humidity and is conservatively assumed to be n =16.

Secondary processing and design strength

Laminated glass

Laminated glass consists of two or more glass plates bonded together by a transparent polymer interlayer, normally polyvinyl butyral (PVB). The nominal thickness of a single PVB foil is 0.38mm and it is normally built-up into two or four layers. Laminating the glass has no observable effect on the crack propagation, but has a significant influence on the post-fracture behaviour.

PVB exhibits viscoelastic behaviour. The flexural behaviour of laminated glass is therefore influenced by the magnitude and duration of loading giving rise to creep of the interlayer and temperature affecting the stiffness of the interlayer. At room temperature, PVB is comparatively soft with an elongation at breakage of more than 200%. At temperatures below 0°C and for short load durations, PVB is sufficiently stiff to transfer the full longitudinal shear from one pane of glass to another. For higher temperatures and long load durations, the shear transfer is greatly reduced. It is common practise to assume some degree of shear transfer (≈ 20%) for short term loading of PVB and to ignore shear transfer for medium and long term loading, although this practise varies from one country to another.

Alternatives to PVB include the recently developed ionoplast interlayers that provide a significantly higher stiffness and tensile strength.

Tempered glass

An effective way of reducing the influence of surface flaws is to treat the glass either thermally or chemically. Both processes rely on establishing a through-thickness residual stress profile where the interior of the glass is in tension and the surface of the glass is in compression. Thermal tempering is the more economical of the two processes and consists of heating the glass to a temperature of 625°C. A through-thickness temperature gradient is set up by cooling the glass rapidly in a quenching plant. As the glass cools to ambient temperature the temperature gradient is transformed to a parabolic stress distribution (Fig 11). The magnitude of surface pre-compression is governed by the rate of cooling leading to two distinct classes of heat treated glass: heat strengthened glass and the stronger fully tempered glass.

In heat treated glass surface cracks may only propagate after the surface pre-compression has been overcome. Equation 5 may therefore be extended to heat treated glass as follows:

$$P_f = 1 - \exp\left(-kA(\sigma_f - f_{rk})^m\right) \quad ...(9)$$

Where f_{rk} is the residual compressive stress on the glass surface and its value is depends the proximity of free edges to the point of interest. In the United States the minimum allowable far-field pre-compression for fully tempered glass is 69MPa[15], whereas in Europe the minimum far-field pre-compression equates to approximately 90MPa[16]. For heat strengthened glass the far-field pre-compression ranges between 24MPa and 52MPa.

The presence of edges, corners and holes distorts the temperature gradient. Recent research suggests that a typical thermally toughened glass plate may be subdivided into four zones. Zones close to an edge, corner or bolt hole, were found to have pre-compressions of 75%, 0% and 68% respectively compared to the far-field regions[17]. Other research suggests that these reductions are less pronounced[18].

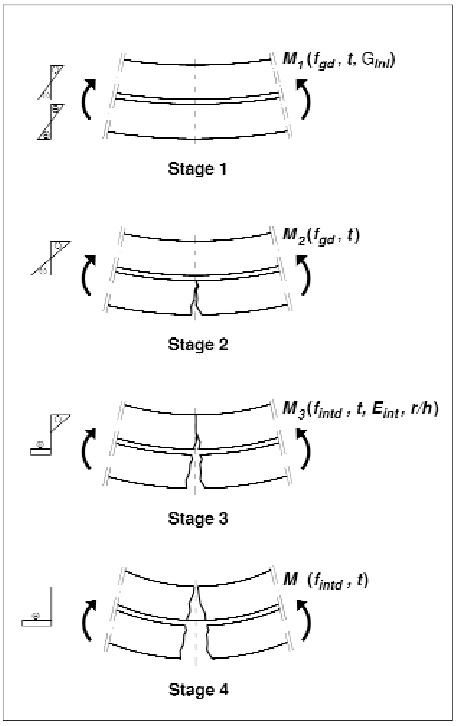
Flaws along the edges of the glass and around the holes caused by the cutting / drilling processes are likely to be larger than the flaws on the glass surface (Fig 12). The flaws may be reduced by polishing, but this may be difficult to achieve in areas with restricted access.

Post-fracture performance

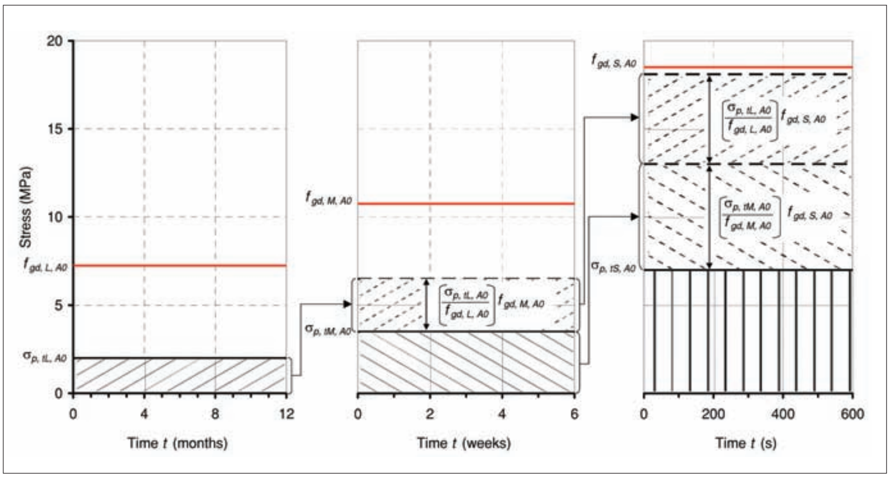
Knowledge of the post fracture performance is essential when considering accidental actions on laminated glass and it is also useful in forensic engineering. Post-fracture behaviour may be characterised by understanding the fragmentation process and the post-fracture collapse mechanics of laminated glass.

Fragmentation

If the stress intensity arising from the applied stresses exceeds the plane strain fracture toughness, dynamic fracture occurs and the crack propagates very rapidly, at approximately 1500ms^{-1}. The resulting fragment size is thought be a function of the strain energy release rate. In such cases dynamic crack branching mechanics



13



14

13 Four stages of post fracture performance for typical a 2-ply laminated glass (contribution of interlayer in stage 1 and stage 2 not shown)

14 Design strength and stress history interaction across long, medium and short term time domains

must be employed to account for the kinetic energy of the advancing crack front. Such formulations are theoretically complex, however simplified empirical relationships may be useful for determining the fragment size and fracture pattern of glass[19]. The empirical relationship shown in equation 10 relates the surface stress at failure σ_f, to the resulting fragment size:

$$\sigma_f - \sigma_{ar,b} = a_b r_b^{-1/2} \qquad ...(10)$$

Where r_b is the one half the crack branch length and the crack branching constant $a_b \approx 2.1\text{MPa m}^{1/2}$ and the apparent residual stress $\sigma_{ar,\,b} \approx 11\text{MPa}$ for annealed glass. In heat-treated glass $\sigma_{ar,\,b}$ should be taken as the surface pre-compression from the heat treatment. Experimental research is currently underway to validate this relationship.

Post-fracture collapse mechanisms

Post-fracture capacity of laminated glass often depends on the composite behaviour between the fragmented glass and the interlayer in laminated glass. A complete analytical method for quantifying the post-fracture strength is still elusive and is a subject of further research. In practise it is therefore necessary to undertake prototype testing to ensure adequate post-breakage capacity.

By extending recent research[20, 21] it is possible to identify four distinct stages of post-fracture flexural resistance (Fig 13). In addition to the interlayer thickness and the glass thickness, the flexural resistance is a function of other factors that vary from one stage to another. In stage 1, both plates of glass are intact and the flexural resistance is a function of the design tensile strength of the bottom glass, f_{gd}, the load duration t, and the shear modulus of the interlayer, G_{int}. In stage 2, the bottom glass plate has fractured and the resistance is largely a function of the design tensile strength of the top glass, f_{gd}, and the load duration t. In stage 3 the top plate has also fractured, but the fragments in the top plate lock together in compression and act compositely with a tensile stress in the interlayer. At this stage the flexural resistance is a function of the load duration t, the design tensile strength of the interlayer f_{intd}, the modulus of elasticity of the interlayer E_{int} and the glass fragment size to glass thickness ratio $2r_b/h$. In stage 4 sufficiently large deformations have taken place and flexural resistance to loads is provided by the design tensile strength of the interlayer f_{intd}, and the load duration t. This stage requires boundary conditions that can translationally retrain the interlayer.

Stage 3 is arguably the most challenging to characterise as it requires knowledge of the fragment size of the top layer of glass. The post-fracture capacity at this stage is contingent on the ability to transfer longitudinal shear stresses between the glass fragments and the interlayer. If the fragment size (i.e. $2r_b$ in equation 10) is small, the contact area between the top glass plate and the individual fragment may be insufficient for mobilising the full bond stresses between the interlayer and the glass, resulting in a reduced post-fracture capacity. It therefore follows that glass that breaks into small fragments, such as fully tempered glass, provides a low post-fracture strength.

Accurate design methods

Structural analysis and modelling

Finite Element Analysis (FEA) is the method of choice for detailed structural analysis of glass elements. Good practise advice on the use of FEA is beyond the scope of the paper and may be found elsewhere[22, 23]. Further specific advice on the use of FEA in glass structures is also available[2, 24]. It is however pertinent to mention two important issues:

- Large lateral deflections, often exceeding the thickness of the glass plate, are a common occurrence in glass structures. In such cases small displacement theory is violated and accurate lateral deflorations and stresses may only be determined by undertaking a geometrically non-linear analysis.
- Bolted connections present several modelling challenges such as the use of contact elements and surfaces releases to simulate the bearing of the bolt on the bolt hole as well as the accurate representation of the rotational stiffness of the connection e.g. whether the bolt is free to rotate as in fully articulated bolts or is semi-rigid as in spring-plate type fixings.

Generalised applied stress and design strength

In order to describe the strength of glass accurately it is useful to start by considering a discrete point (x, y) on the surface of a glass element which has a pre-compression f_{rk} from the tempering process acting on it. Point (x, y) is subjected to a time-varying major principle stress history $\sigma_1(t)$ applied for a duration of t_f. arising from the design load F_d. By extending equation 7 to account for the surface pre-compressions from the temperring

Glass type	Maximum span / thickness	
	Vertical	Sloping or Horizontal
Annealed glass	150	100
Fully tempered glass	200	150
Laminated annealed glass	150	100
Laminated tempered glass	150	100

Table 4 Approximate span/thickness ratios for glass simply supported along two or four edges and subjected to lateral loading[30]

Glass type	Approximate Strength f_{adm} (MPa)	
	Vertical	Overhead
Annealed glass	18	12
Fully tempered glass	50	50
Laminated glass	22.5	15 (25*)

* Lower pane strength in accidental design situation i.e. after upper pane is broken

Table 5 Approximate strength[31, 32]

process, we may obtain an equivalent constant stress σ_{t0} applied for a reference time t_0 that causes the equivalent stress corrosion of the surface as the original time-varying stress:

$$\sigma_{t0} = \left[\frac{1}{t_0}\int_{t_r}^{t_f}\left(\sigma_1(t) - \frac{f_{rk}}{\gamma_{mv}}\right)^n dt\right]^{1/n} \quad where\left(\sigma_1(t) - \frac{f_{rk}}{\gamma_{mv}}\right) > 0 \qquad ...(11)$$

This transformed tensile stress however, represents the contribution of one discrete point (x, y) to the failure of the glass element. It is necessary to summate the contribution of all the points on the glass surface by adopting the size effect described in equation 6. Equation 11 therefore becomes:

$$\sigma_{p,t0,A0} = \left\{\frac{1}{A_0}\int_{area}\left[\frac{1}{t_0}\int_{t_r}^{t_f}\left(\sigma_1(t) - \frac{f_{rk}}{\gamma_{mv}}\right)^n dt\right]^{m/n} dA\right\}^{1/m} \qquad ...(12)$$

This equation transforms the real-world applied stress which varies over time and across the surface area of the glass element into an equivalent uniformly distributed stress $\sigma_{p,t0,A0}$ applied constantly for a reference time t_0.

The design strength of glass $f_{gd,t0,A0}$ may also be expressed in similar equivalent terms i.e. a constant uniform resistance for a reference time t_0:

$$f_{gd,t0,A0} = \frac{k_{mod} f_{gk}}{\gamma_{mA}} \qquad ...(13)$$

It is now possible to compare the design strength and the equivalent uniform stress to ensure that:

$$f_{gd,t0,A0} \geq \sigma_{p,t0,A0} \qquad ...(14)$$

Applied stress and design strength in practice

The accurate approach described in the preceding section has two major drawbacks in practice, namely that: (a) a continuous stress history function $\sigma(t)$ is generally unsuitable for describing real-world load combinations and; (b) the computation of the equivalent uniformly distributed stress $\sigma_{p,t0,A0}$ shown in equation 12 is unattractive for manual calculations. These disadvantages may be overcome by applying short (F_{dS}), medium (F_{dM}) and long term (F_{dL}) combinations to the glass element generated from equation 1. The surface stresses resulting from these design loads can be converted to equivalent uniform stresses for short ($\sigma_{p,tS}$), medium ($\sigma_{p,tM}$) and long ($\sigma_{p,tL}$) reference load durations as follows:

$$\sigma_{p,tS,A0} = \left[\frac{1}{A_0}\int_{area}\left(\sigma_{1,tS} - \frac{f_{rk}}{\gamma_{mv}}\right)^m dA\right]^{1/m} \quad where\ \left(\sigma_{1,tS} - \frac{f_{rk}}{\gamma_{mv}}\right) > 0 \qquad ...(15)$$

$$\sigma_{p,tM,A0} = \left[\frac{1}{A_0}\int_{area}\left(\sigma_{1,tM} - \frac{f_{rk}}{\gamma_{mv}}\right)^m dA\right]^{1/m} \quad where\ \left(\sigma_{1,tM} - \frac{f_{rk}}{\gamma_{mv}}\right) > 0 \qquad ...(16)$$

$$\sigma_{p,tL,A0} = \left[\frac{1}{A_0}\int_{area}\left(\sigma_{1,tL} - \frac{f_{rk}}{\gamma_{mv}}\right)^m dA\right]^{1/m} \quad where\ \left(\sigma_{1,tL} - \frac{f_{rk}}{\gamma_{mv}}\right) > 0 \qquad ...(17)$$

where $\sigma_{1\ tS}$, $\sigma_{1\ tM}$, $\sigma_{1\ tL}$ represent the major principal stresses resulting from elastic analyses of the glass element subjected to the load F_{dS}, F_{dM} and F_{dL} respectively.

Equations 15, 16 and 17 are still quite unattractive for manual computation. It is however relatively simple to setup a computer algorithm to compute these stresses automatically. This approach is particularly convenient when the glass element is modelled in FEA software where the algorithm is in the form of a script that extracts the relevant principle stresses from the individual finite elements and summate their contribution over the whole glass member[25, 26, 27].

A less accurate, but safe approximation is to adopt a maximum stress approach whereby the largest major principal stress acting on the surface of the glass is identified and is assumed to be acting over the entire surface of the glass element. In doing so, equations 15, 16 and 17 reduce to:

$$\sigma_{p,tS,A0} = \sigma_{\max,tS} - \frac{f_{rk}}{\gamma_{mv}} \qquad ...(18)$$

$$\sigma_{p,tM,A0} = \sigma_{\max,tM} - \frac{f_{rk}}{\gamma_{mv}} \qquad ...(19)$$

$$\sigma_{p,tL,A0} = \sigma_{\max,tL} - \frac{f_{rk}}{\gamma_{mv}} \qquad ...(20)$$

The short, medium and long term stresses from either the accurate equivalent stress approach (i.e. equations 15 to 17) or the approximate maximum stress approach (i.e. equations 18 to 20) must be compared to the respective time resolved tensile design strength of glass. The design strength for short, medium and long term loads $f_{gd,tS,A0}$, $f_{gd,tM,A0}$ and $f_{gd,tL,A0}$ may be calculated from equation 13 where k_{mod} = 0.74, 0.43 and 0.29 for short, medium and long term loads respectively (cf. table 2). Adequate resistance to short, medium and long term loads is ensured by satisfying the following stress-history interaction equation:

$$\frac{\sigma_{p,tS,A0}}{f_{gd,tS,A_0}} + \frac{\sigma_{p,tM,A0}}{f_{gd,tM,A_0}} + \frac{\sigma_{p,tL,A0}}{f_{gd,tL,A_0}} \leq 1 \qquad ...(21)$$

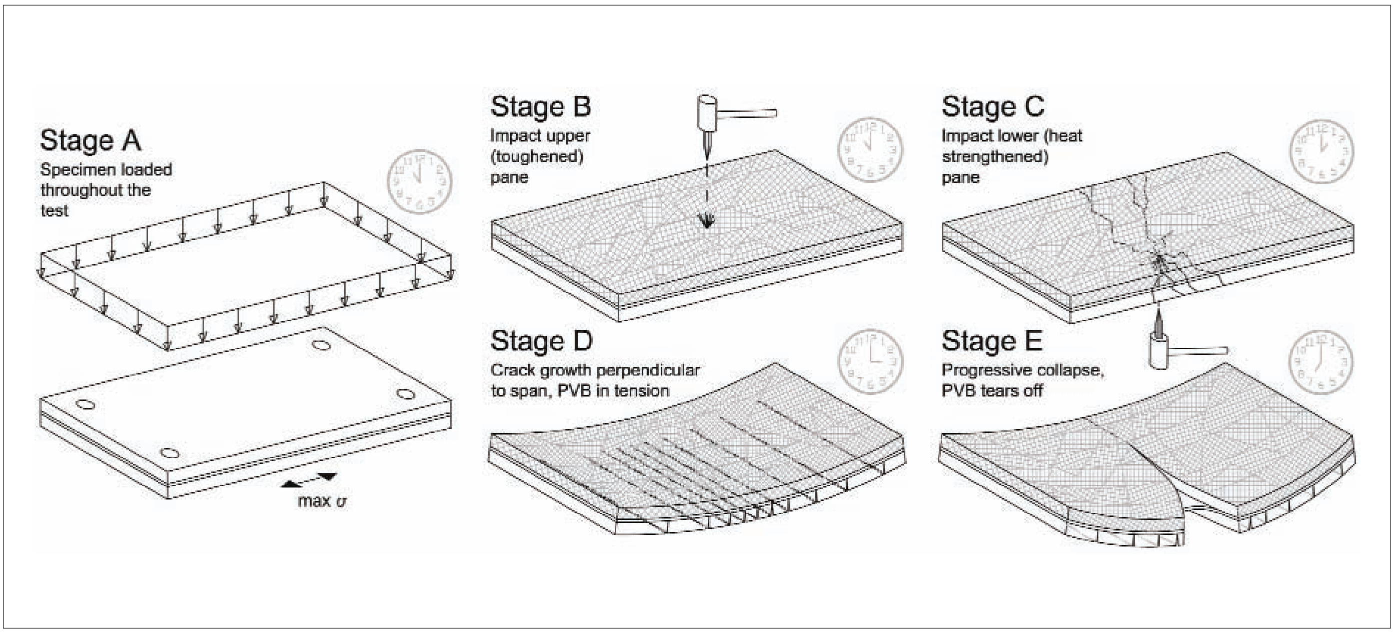
The three fractions on the left hand side of equation 23 are in effect the contributions to failure (i.e. stress corrosion) caused by the short, medium and long term loads respectively.

Fig 14 illustrates how the stress-history interaction expressed in equation 21 accounts for the loss in strength across the three time domains. The figure shows a generalised case where a structural glass element is subjected to long term (12 month) load, followed by a medium term (4 week) load and finally by a short term (5 minute) load. These loads result in equivalent uniform stresses on the glass surface $\sigma_{p,tL,A0}$, $\sigma_{p,tS,A0}$, and $\sigma_{p,tM,A0}$ in the long medium and short time domains respectively. The dashed hatched areas in Fig 14 represent the stress corrosion caused by the preceding load events. Ignoring the stress history described by this interaction equation leads to an overestimation of design strength and is therefore unsafe.

Rules of thumb

It is unrealistic to expect engineers to use the rigorous calculations described above for preliminary sizing and other quick checking purposes required throughout the design process.

There are relatively few approximate methods for the structural analysis of glass elements. Two useful sources of information for



15 Test sequence for residual post-fracture capacity of overhead glass

15

determining the surface stresses and deflections are approximate formulae for plates undergoing large deformations[28] and the charts for determining stress concentrations around bolt holes[29].

Approximate clear spans to thickness ratios and values for the approximate tensile strength of glass f_{adm} are shown in Table 4 and Table 5 respectively. The latter are extracted from the German TRAV[31] and the TRLV[32] technical guidelines and account for specific conditions of stress history, environmental condition, and surface area.

Design assisted by testing

Despite advances in analytical and computational methods there are several instances when calculations require further validation by prototype testing. This applies where confidence in the design methods is low such as in novel glass structures or where analytical methods are still under development such as in the design for impact and blast loads and for determining post-fracture capacity.

General guidelines for design assisted by testing are provided in Annex D of EN 1990:2002[3], but it is the engineer's responsibility to specify a suitable and project-specific test regime. When testing glass structures it is important to bear the following in mind:

- Nominally identical test specimens often produce a wide scatter of strength data. It is therefore advisable to have as large a sample size as possible. Anything below 10 specimens is unlikely to be statistically significant.
- The surface condition of the specimens tested must be representative of the that expected during their service-life.
- Glass is particularly sensitive to stress history. During testing, the loading rate should be carefully recorded. The strength data obtained form each specimen should then be converted to a common reference stress duration (cf. equation 7) before any statistical analysis is undertaken.
- Glass strength is only affected by surface condition, surface area, humidity and load duration after the thermally induced pre-compression has been overcome by the applied tensile stresses. (cf. equation 11).

Impact and blast testing

Impact testing varies from country to country, but there are generally two categories of impact: (a) soft body impact used to assess the performance of balustrades, walls etc. to simulate human impact. (b) hard body impact used for overhead glazing to simulate the dropping of hard objects onto the glass. Bomb blast testing is undertaken by means of arena testing in a secure range testing site or by using shock tube equipment. A more detailed review of blast test requirements in Europe and the United States is provided in Haldimann *et al*.[2.]

Testing for post-fracture performance

The assessment of post-breakage performance generally involves the application of a static load for a given duration after the glass has failed. This experimental method is a measure of the robustness of the glass and it is often possible to combine this test with impact strength tests.

There is a lack of standardised testing procedures for post-breakage performance. A typical procedure used in Europe is shown in Fig 15 and consists of: (a) load the specimen to half the service load or 0.5kPa, whichever is greater; (b and c) with the load still applied, fracture all the plates of the laminated glass panel by means of a centre punch and a hammer (d and e) monitor the performance for 24h. The laminated glass panel is deemed to pass the test if the specimen remains attached to the supports and no dangerous glass fragments fall out during the test.

Conclusion

The accurate design of glass structures is a non-trivial task where the engineer has to account for the several factors that affect glass strength. In doing so the engineer must deploy his / her knowledge of fracture mechanics, geometrically non-linear FEA and statistical analysis. It is however unrealistic to expect the engineer to undertake these demanding tasks throughout the design process.

This paper provides an overview of various design methods ranging from the basis of accurate design methods, to rules of thumb and prototype testing and it provides some guidance on when the various methods should be deployed. Furthermore, the stress-history interaction equation presented in this paper will enable engineers to account for multiple load durations in glass design.

The paper is not exhaustive in nature for two reasons. Firstly the topics discussed in the paper are quite vast in their own right; readers should therefore refer to the extensive list of references at the end of this paper. Secondly, structural glass is still in its infancy and several design methods are the subject of on-going research and development.

Acknowledgements

Thanks are due to Dr Mathias Haldimann and Dr Andreas Luible who have contributed to this paper. Thanks are also due to Benjamin Beer for his input in the section on Design Assisted by

Testing, and to the various undergraduate and post-graduate students in the Glass and Façade Technology research group at Cambridge, whose input is gratefully acknowledged.

Notation

a	crack depth or one half crack length
a_b	crack branching constant (= 2.1MPa m$^{1/2}$)
A	surface area exposed to tensile stress
A_0	reference surface area (= 1m^2).
A_d	design value of an accidental action
E_i	modulus of elasticity of interlayer
f_{adm}	approximate strength of glass incorporating approximations for stress history, environmental condition, surface area and thermal prestress
f_{gd}	design strength of glass
$f_{gd,\,t}$	design strength of glass for given load duration t
f_{gk}	characteristic short-term tensile strength of glass (≈ 45MPa)
f_{id}	design tensile strength of interlayer
f_{rk}	magnitude of residual surface stress due to tempering (also known as tempering prestress)
F_d	design value of the combination of actions
G	value of permanent actions (e.g. self-weight load, permanent equipment).
G_I	shear modulus of interlayer
h	glass thickness
k	surface strength parameter describing Weibull distribution
k_{mod}	stress corrosion ratio (also known as stress duration factor)
m	surface strength parameter describing Weibull distribution
P_f	probability of failure
$Q_{k,1}$	characteristic value of the leading variable action (e.g. imposed load on floor, wind, snow)
$Q_{k,i}$	characteristic value of the accompanying variable action (e.g. wind, snow)
r_b	one half crack branch length
t_f	time to failure
t_0	reference time period
Y	flaw geometry factor
ψ	radius of curvature
$\psi_{0,i}$	factors for combination value of accompanying variable actions
ψ_1	factor for frequent value of a variable action
$\psi_{2,i}$	factor for quasi-permanent value of a variable action
γ_G	partial factor for permanent actions, also accounting for model uncertainties and dimensional variations
γ_{mA}	material partial factor for annealed glass
γ_{mV}	material partial factor for tempering prestress
γ_Q:	partial factor for variable actions, also accounting for model uncertainties and dimensional variations
$\sigma_{ar,b}$	apparent residual stress (≈ 11MPa for annealed glass; ≈ f_{rk} for tempered glass))
σ_1	major principal stress
σ_n	nominal tensile stress normal to the crack plane
σ_f	failure stress
σ_p	equivalent uniform applied stress
$\sigma_{p,t0}$	equivalent uniform stress applied for duration t_0
σ_{t0}	t_0 equivalent stress.

References

1 IStructE.: *Structural use of glass in buildings*, The Institution of Structural Engineers, London, December 1999

2 Haldimann, M., Luible, A., and Overend, M.: 'Structural use of glass', *Structural Engineering document No.10*, International Association of Bridge and Structural Engineers (IABSE), 2008

3 EN 1990: *Eurocode – Basis of structural design,* CEN, 2002

4 prEN 13474-1: *Glass in building – Design of glass panes – Part 1: General basis of design*, CEN, 2007

5 EN 1991-1-7: *Eurocode 1. Actions on structures. General actions – Accidental actions*, CEN, 2006

6 EN 572-1: *Glass in building – Basic soda lime silicate glass products – Part 1: Definitions and general physical and mechanical properties*, CEN, 2004

7 EN 1748-1-1: *Glass in building – Special basic products – Borosilicate glasses – Part 1-1: Definitions and general physical and mechanical properties*, CEN, 2004

8 Griffith A. A.: 'The phenomena of rupture and flow of solids', *Theoret. Trans. Royal Society of London*,1920, 221, p163-179,

9 Irwin, G.: 'Analysis of stresses and strains near the end of a crack traversing a plate', *J. App. Mech*.,1957, 24, p361–364,

10 Haldimann, M.: 'Fracture strength of structural glass elements – analytical and numerical modelling, testing and design'. *Thèse EPFL No 3671*, Ecole polytechnique fédérale de Lausanne (EPFL), 2006

11 Weibull, W.: 'A statistical distribution function of wide applicability', *J. App. Mech*., September1951,18, p293–297

12 Behr R. A., Karson M. J. and Minor, J. E.: 'Reliability analysis of window glass failure pressure data, *Structural Safety*, 1991, 11, p43-58,

13 Sedlacek, G., Blank, K., Laufs, W. and Güsgen, J.: *Glas im Konstruktiven* Ingenieurbau Ernst & Sohn, Berlin, 1999

14 Grenet, L.: Mechanical strength of glass, *Enc. Industr. Nat. Paris*, 1899, 5/4, p838–848

15 Brown, W. G.: 'A practicable formulation for the strength of glass and its special application to large plates', *Publication No. NRC 14372*, National Research Council of Canada, Ottawa, 1974

16 ASTM C 1048-04. *Standard Specification for Heat-Treated Flat Glass – Kind HS, Kind FT Coated and Uncoated Glass*. American Society for Testing Materials, 2004

17 EN 12600:2002. *Glass in building – Pendulum test – Impact test method and classification for flat glass*, CEN

18 Laufs, W.: 'Ein Bemessungskonzept zur Festigkeit thermisch vorgespannter Gläser', *PhD. thesis*, RWTH Aachen / Shaker Verlag, 2000

19 Schneider, J.: 'Festigkeit und Bemessung punktgelagerter Gläser und stossbeanspruchter Gläser'. *PhD. thesis*, TU Darmstadt, Institut für Statik, 2001

20 Overend, M., De Gaetano, S. and Haldimann, M.: 'Diagnostic interpretation of glass failure'. *Struct. Eng. Int.*, 2007, 17/2, p151–158

21 Kott, A. and Vogel, T.: 'Safety of laminated glass structures after initial failure', *Struct. Eng. Int.*, 2004, 14/2, p134–138

22 Kott, A. and Vogel, T.: 'Structural behaviour of broken laminated safety glass', *Proc., Glass in Buildings 2*; April 2005, Bath UK

23 Cook, R. D.: *Finite Element Modelling For Stress Analysis*, Wiley, New York, 1995. ISBN 0471107743

24 Zienkiewicz, O. C. and Taylor, R. L.: *The Finite Element Method* (3 volumes). Butterworth-Heinemann, 2005, ISBN 0-750-66431-2

25 Overend, M. and Maniatis, I.: 'The influence of finite element modelling parameters on the stress distribution in bolted glass connections', *Proc. 11th Int. Conf. Civ., Struct. and Environ. Eng. Comp*, Malta, 2007

26 Overend, M.: 'The appraisal of structural glass assemblies'. *PhD. thesis*, University of Surrey, March 2002

27 Overend, M., Parke, G. A. R. and Buhagiar, D.: 'A general crack growth model for predicting failure in glass, 2007, *J. Struct. Eng*., ASCE, 133/8, p1146-1155

28 Young, W. C. and Budynas, R. G.: *Formulas for Stress and Strain*, McGraw-Hill Publishing Co., 2002, ISBN 0-071-21059-8

29 Pilkey, W. D.: *Peterson's Stress Concentration Factors*, 2nd Ed., Wiley, New York, 1997, ISBN 0471538493

30 Colvin, J.: 'Look – No frames!' *Proc. Glass Processing Days 2003*, 15-18 June 2003, Tampere, Finland, pp. 208–210

31 *TRAV 2003. Technische Regeln für die Verwendung von absturzsichernden Verglasungen (TRAV)*. Technical Report, Deutsches Institut für Bautechnik (DIBt), January 2003

32 *TRLV 2006. Technische Regeln für die Verwendung von linienförmig gelagerten Verglasungen (TRLV).* Technical Report, Mitteilungen des Deutschen Instituts für Bautechnik (DIBt), Berlin, 2006